

ANALYSIS OF FUNDAMENTAL AND HIGHER ORDER SOLITONIC PROPAGATION IN OPTICAL FIBER

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ABSTRACT

The objective of this paper is to study the basic analytical propagation of fundamental and higher order solitons in the optical fiber. The characteristic feature of soliton formation occurs due to the balance between Self-Phase Modulation (SPM) and Group Velocity Dispersion (GVD). We simulate the fundamental and higher order soliton in time and frequency domain to investigate the propagation of solitonic pulses in nonlinear optical fibers.

KEYWORDS: Solitons, Optical Fibers, Numerical Analysis

1. INTRODUCTION

The word *soliton* refers to special kinds of wave packets that can propagate undistorted over far off distances. Solitons have been discovered in many branches of Physics. In the context of optical fibers, solitons are not only of fundamental interest but also have practical applications in the field of fiber-optic communications. James Scott Russel was the first scientist who observed a soliton wave in 1834 when he accidentally noticed in the narrow water canal a smoothly shaped water heap that to his surprise was able to propagate in the canal without an apparent change in its shape a few kilometers along. The fact of propagation of this solitary wave was not understood for a long time until appropriate mathematical model was conceived in the 1960's together with a way of solving nonlinear equation with the help of inverse scattering method.

The phenomenon of modulation instability reveals that propagation of a continuous-wave (CW) beam inside optical fibers is inherently unstable because of the nonlinear phenomenon of SPM and leads to formation of a pulse train in the anomalous dispersion regime of optical fibers.

2. MATHEMATICAL MODELING AND PROPAGATION OF THE SOLITARY WAVE IN OPTICAL FIBER

The propagation of light can be described, more precisely, with Maxwell mathematical equations. When equations for magnetic and electric fields are combined together we find:

$$\nabla^2 \vec{E} - \frac{1}{c^2} * \frac{\partial^2}{\partial t^2} \vec{E} = \frac{1}{\epsilon_0 c^2} * \frac{\partial^2}{\partial t^2} \vec{P} \quad (1)$$

where c is the speed of light in the vacuum and ϵ_0 is the vacuum permittivity. The induced polarization P consists of two parts:

$$\vec{P}(\vec{r}, t) = \vec{P}_L(\vec{r}, t) + \vec{P}_{NL}(\vec{r}, t) \quad (2)$$

where $P_L(r,t)$ and $P_{NL}(r,t)$ are related to electric field by relations:

$$\vec{P}_L(\vec{r}, t) = \epsilon_0 \int_{-\infty}^{+\infty} \chi^{(1)}(t-t') \cdot \vec{E}(\vec{r}, t') dt' \quad (3)$$

$$\vec{P}_{NL}(\vec{r}, t) = \epsilon_0 \iiint_{-\infty}^{+\infty} \chi^{(3)}(t-t_1, t-t_2, t-t_3) * \vec{E}(\vec{r}, t_1) \vec{E}(\vec{r}, t_2) \vec{E}(\vec{r}, t_3) dt_1 dt_2 dt_3 \quad (4)$$

where $\chi^{(1)}$ and $\chi^{(3)}$ are the first and third order susceptibility tensors.

For better understanding of a soliton pulse propagation in optical fiber, it necessary to set up our modelling on the mathematical expression (1). We suppose that a solution for electric field E has a form [1]:

$$E(r,t) = A(Z,t) F(X,Y) \exp(i\beta_0 Z) \quad (5)$$

where $F(X,Y)$ is transverse field distribution that corresponds to the fundamental mode of single mode fiber.

$A(Z,t)$ is along propagation axis Z and on time t dependent amplitude of the mode. After some mathematical manipulations we can come to the equation that governs pulse propagation in optical fibers [1]:

$$\frac{\partial A}{\partial Z} + \beta_1 \frac{\partial A}{\partial t} + \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial t^2} = i\gamma |A|^2 A \quad (6)$$

The parameters β_1 and β_2 include the effect of dispersion to first and second orders, respectively. Physically, $\beta_1 = 1/v_g$, where v_g is group velocity associated with the pulse and takes into account the dispersion of group velocity. For this reason, β_2 is called the group velocity dispersion (GVD) parameter. Parameter γ is nonlinear parameter that takes into account the nonlinear properties of a fiber medium. Parameter β_1 is, in real case, always positive but on the other hand parameters λ_{ZD} and γ can be, in some specific case, either positive or negative. The parameter β_1 is closely associated in practice with better known parameter called dispersion parameter - D (ps/nm/km). The relation between them is in the form [1]:

$$D = \frac{d}{d\lambda} \left(\frac{1}{v_g} \right) = -\frac{2\pi c}{\lambda^2} \beta_2 \quad (7)$$

As we know, dispersion parameter D is a monotonically increasing function of wave-length, crossing a zero point at wave-length λ_{ZD} , which is called a zero chromatic dispersion wave-length. If a system operates with wave-lengths above λ_{ZD} , where D is positive, β_2 must be negative and a fiber is said to work in anomalous dispersion mode. If a fiber is operated below λ_{ZD} , the D is negative and β_2 must be positive. In this case a fiber is said to operate in normal dispersion mode. As regards the nonlinear parameter γ , it can generally be either positive or negative, depending on the material of the wave guide.

For silica fiber parameter γ is positive but for some other materials it can be negative. More specifically, equation (6) has only two solitons - either dark or bright solitons. The bright soliton corresponds to the light pulse but dark soliton is rather a pulse shaped dip in CW light background. In other words, the dark soliton is in fact negation of the bright soliton. While there is maximum of light in the bright soliton, the dark soliton has minimum of the light. The bright soliton can propagate in only such a waveguide where there is either the positive nonlinearity parameter and fiber in anomalous dispersion regime or the negative nonlinear parameter but in normal dispersion regime.

3. SIMULATION AND RESULTS

Equation (6) can be defined in the form:

$$i \frac{\partial u}{\partial z} - \frac{s}{2} \frac{\partial^2 u}{\partial \tau^2} \pm |u|^2 u = 0 \tag{8}$$

Through a simple conversion:

$$\tau = (t - \beta_1 Z) / T_0, \quad z = Z / L_D, \quad u = \sqrt{|\gamma| L_D} A \tag{9}$$

where T_0 is pulse width and $L_D = T_0^2 / |\beta_2|$ is the dispersion length. Through inverse scattering method it is revealed that the solution of above mentioned equation has a form:

$$u(z, \tau) = N * 2 / (e^\tau + e^{-\tau}) * e^{iz/2} = N \operatorname{sech}(\tau) e^{iz/2} \tag{10}$$

If N is integer, it represents the order of the soliton pulse. A very interesting situation comes when $N=1$.

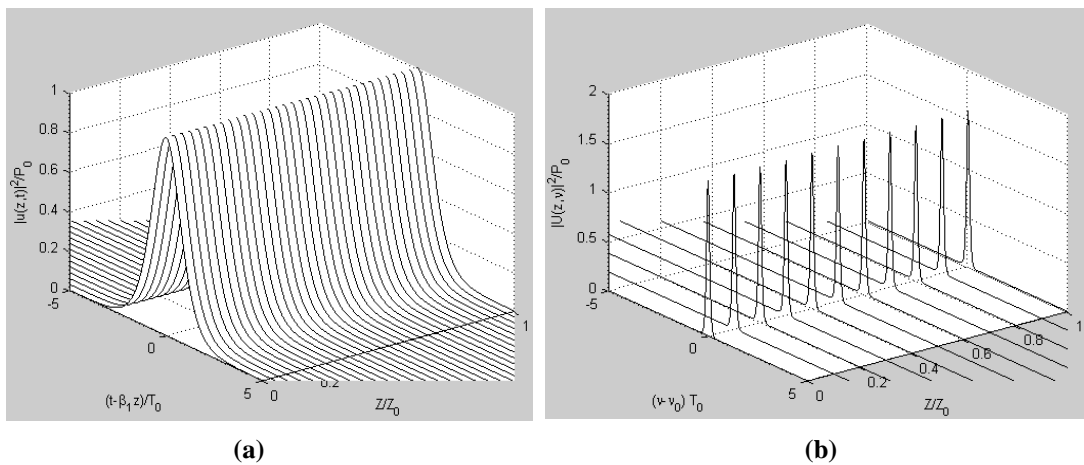


Figure 1: Evolution of the Pulse Spectrum for N=1: (a) Time Domain and (b) Frequency Domain

In the case of first order soliton, the pulse does not change its shape at all as it propagates in optical fiber. In contrast when N is higher than one, pulse shape is not stable and changes periodically with soliton period $Z_0 = (\pi / 2) L_D$. At the end of every period Z_0 the soliton resembles its initial simple pulse shape.

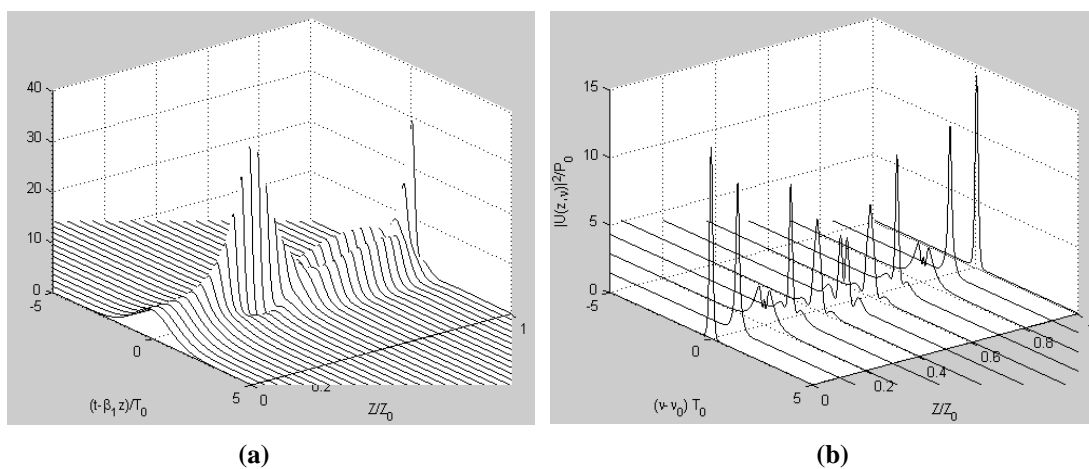


Figure 2: Evolution of the Pulse Spectrum for N=3: (a) Time Domain and (b) Frequency Domain

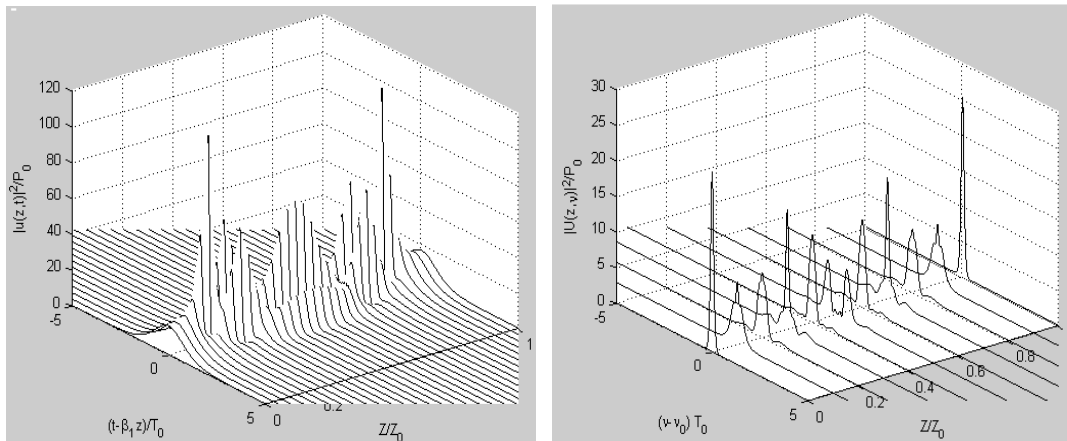


Figure 3: Evolution of the Pulse Spectrum for N=4: (a) Time Domain and (b) Frequency Domain

It is evident that for telecommunication purposes the soliton of first order is most suitable because in this application it is necessary to keep a pulse shape stable. N defines the order of soliton as under:

$$N = T_0 \sqrt{\frac{\gamma P_0}{\beta_2}} \quad (11)$$

where T_0 corresponds to input pulse width, P_0 is pulse peak power. β_2 takes into account group velocity dispersion and γ is nonlinear parameter of the fiber material.

4. CONCLUSIONS

In this paper a numerical analysis based on simulations has been employed to elaborate the propagation of solitonic pulses in nonlinear optical fibers. The Analysis have been carried out in time and frequency domain to fully study the nonlinear effect. Several discussions based on numerical results have been conducted.

REFERENCES

1. AGRAWAL, Govind P. *Applications on Nonlinear Fiber Optics*. San Diego: Academic Press, 2001. 458 s. ISBN 0-12-045144-1.
2. KIVSHAR, Yuri S., AGRAWAL, Govind P. *Optical Solitons: From Fibers to Photonic Crystals*. San Diego: Academic Press, 2003. 540 s. ISBN 0-12-410590-4.
3. PORSEZIAN, K., KURIAKOSE, V.C. *Optical Solitons: Theoretical and Experimental Challenges*. 1st edition. Berlin: Springer, 2003. 406 s. ISBN 3-540-00155-7.
4. GAGLIARDI, Robert M., KARP, Sharman. *Optical Communications*. 2nd edition. New York: Wiley, 1995. 347 s. ISBN 0-471-54287-3.